# Model STAR (Smooth Transition Autoregressive)

V modeloch TAR prechádzame z jedného režimu do druhého skokom - okamžite. Nahradením nespojitej funkcie $I(q_t > c)$ hladkou funkciou $G(q_t, \gamma, c)$, ktorá s rastúcim $q_t$ spojito prechádza od 0 do 1 docielime to, že prechod z jedného režimu do druhého je plynulý. Tento typ modelu sa označuje ako model STAR (Smooth Transition Autoregressive):

$$X_t = \Phi'_1 Y_t + (\Phi_2 - \Phi_1)' Y_t G(q_t, \gamma, c) + \varepsilon_t$$

kde $p = \max(p_1, p_2)$, $Y_t = (1, X_{t-1}, \ldots, X_{t-p})'$, $\Phi_i = (\varphi_{i,0}, \varphi_{i,1}, \ldots, \varphi_{i,p})'$, $i = 1, 2$;

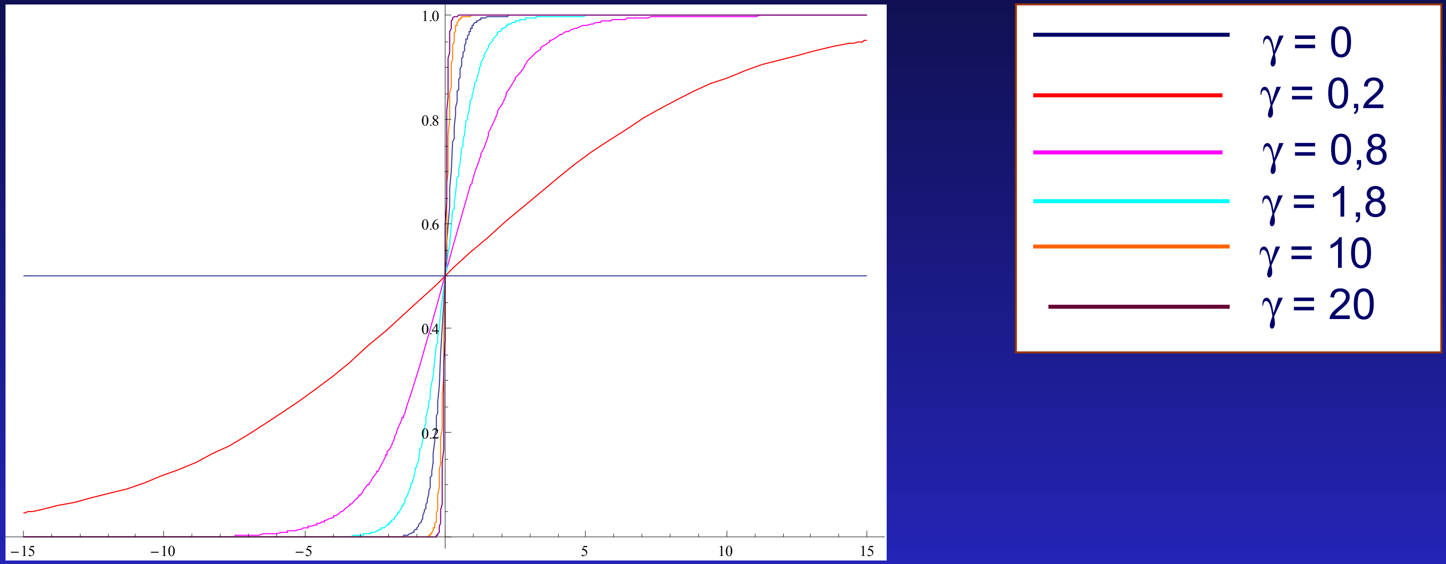
$\{\varepsilon_t\}$ je i.i.d. s nulovou strednou hodnotou a rozptylom $\sigma_\varepsilon^2$.

Funkcie $G(q_t, \gamma, c)$ sa nazýva **prechodová funkcia**. Parameter vyhladzovania $\gamma$ určuje ***hladkosť*** zmeny hodnôt prechodovej funkcie a teda aj rýchlosť prechodu z jedného režimu do druhého.

Často používanou prechodovou funkciou je **logistická** funkcia:

$$G(q_t, \gamma, c) = \frac{1}{1 + e^{-\gamma(q_t - c)}}, \quad \gamma > 0$$

Modely s logistickou prechodovou funkciou sa nazývajú **LSTAR**



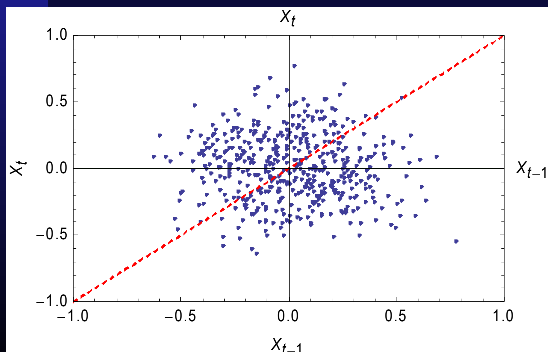

*c* = 0

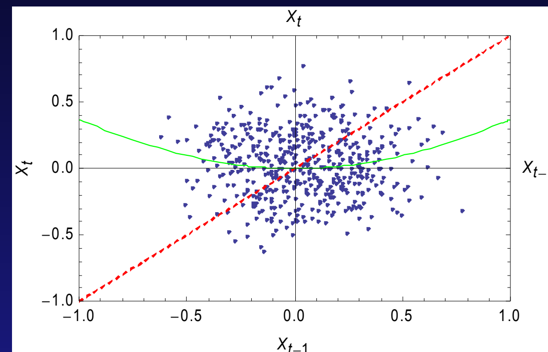
Ak je hodnota $\gamma$ veľmi veľká ($\gamma \to \infty$), prechod je takmer okamžitý a logistická funkcia sa správa veľmi podobne ako indikačná funkcia I[A] . Ak hodnota parametra $\gamma \to 0$, hodnota logistickej funkcie nadobúda konštantnú hodnotu ½ , $G(q_t;\ 0,\ c) = ½$ a model sa redukuje na lineárny model.

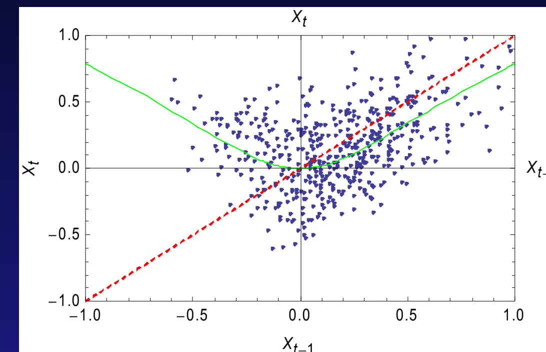
# LSTAR v závislosti na $\gamma$

$$X_t = (\varphi_{1,0} + \varphi_{1,1} X_{t-1})(1 - G(X_{t-1}, \gamma, c)) + (\varphi_{2,0} + \varphi_{2,1} X_{t-1}) G(X_{t-1}, \gamma, c) + \varepsilon_t$$

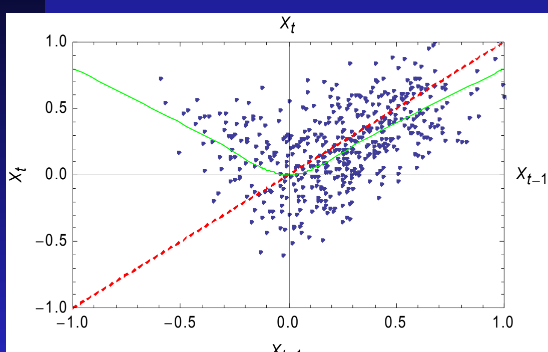
$c = 0; \varphi_{1,0} = \varphi_{2,0} = 0; \quad \varphi_{1,1} = -0.8; \quad \varphi_{2,1} = 0.8; X_0 = 0;$

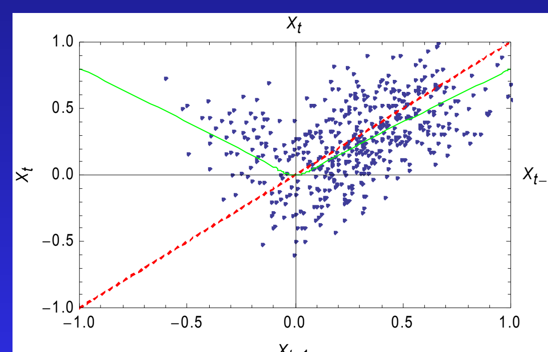


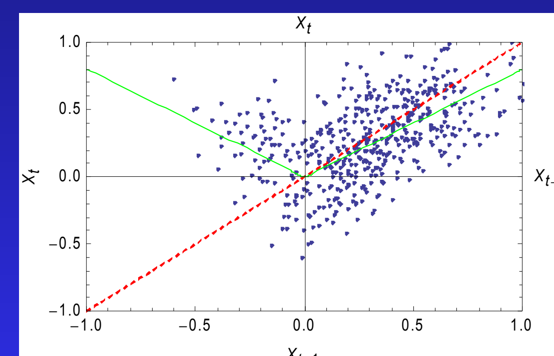




**$\gamma = 0;$** **$\gamma = 0.5;$** **$\gamma = 5;$**



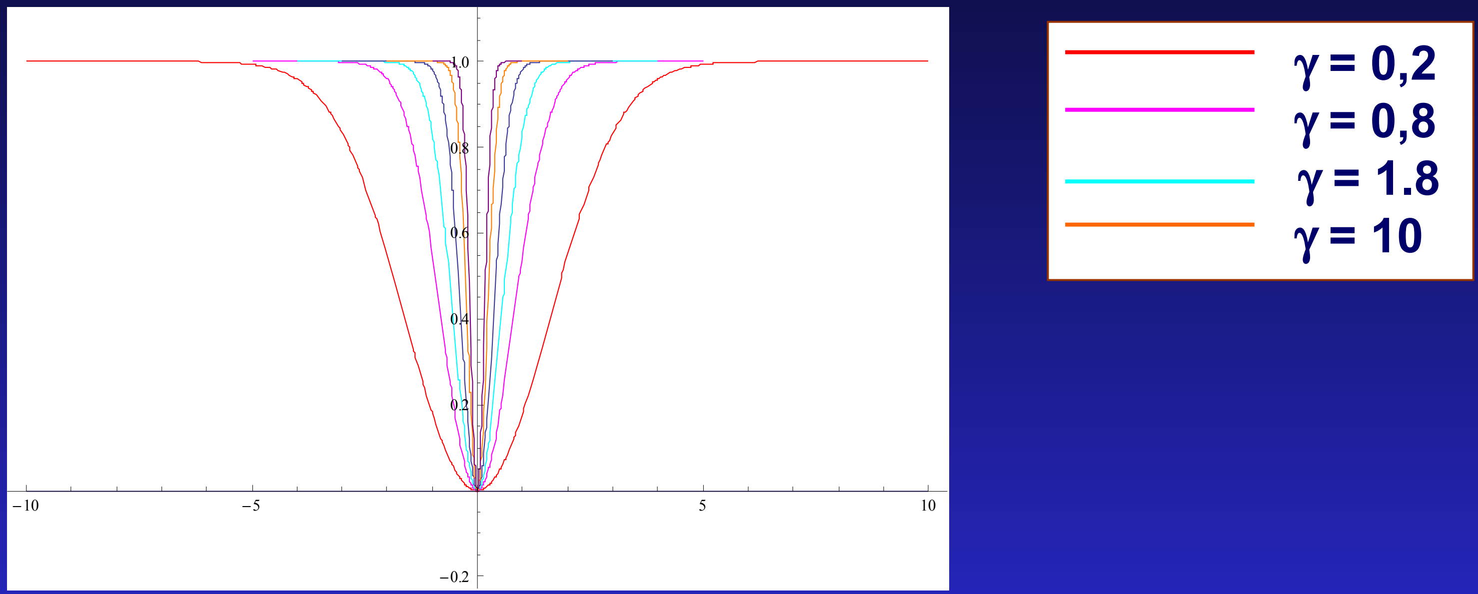




**$\gamma = 10;$** **$\gamma = 20;$** **$\gamma = 50;$**

V ekonómii sa často používa exponenciálna funkcia:

$$G(\boldsymbol{q}_t, \gamma, \boldsymbol{c}) = 1 - e^{-\gamma(\boldsymbol{q}_t - \boldsymbol{c})^2}, \quad \gamma > 0$$

Modely s exponenciálnou prechodovou funkciou sa nazývajú ESTAR



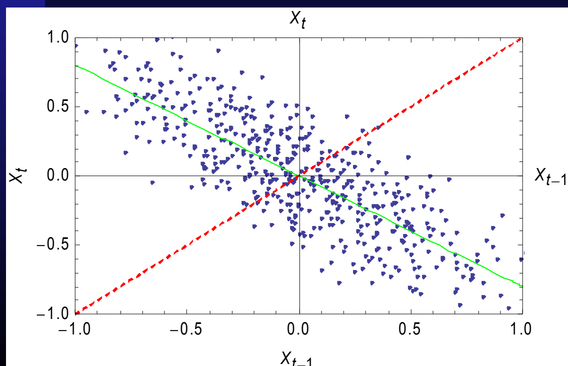

$c = 0$

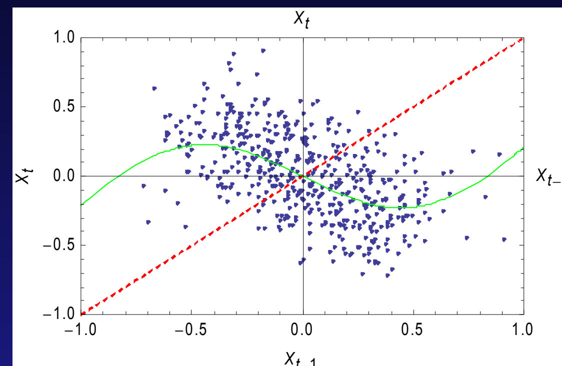
**Ak je hodnota γ veľmi veľká ($\gamma \to \infty$), $G(q_t, \gamma, c) \to 1$. Ak hodnota parametra $\gamma \to 0$, hodnota logistickej funkcie nadobúda konštantnú hodnotu 0, $G(q_t; 0, c) = 0$. V obidvoch prípadoch sa model redukuje na lineárny model.**

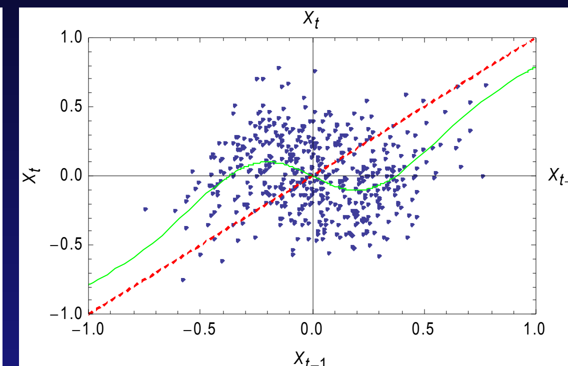
# ESTAR v závislosti na $\gamma$

$$X_t = (\varphi_{1,0} + \varphi_{1,1} X_{t-1})\,(1 - G(X_{t-1}, \gamma, c)) + (\varphi_{2,0} + \varphi_{2,1} X_{t-1})\,G(X_{t-1}, \gamma, c) + \varepsilon_t$$

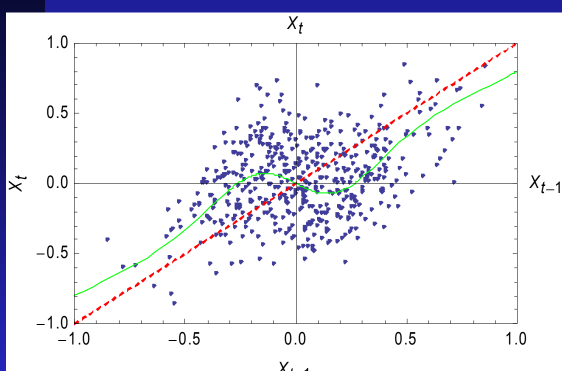
$c = 0;\ \varphi_{1,0} = \varphi_{2,0} = 0;\quad \varphi_{1,1} = -0.8;\quad \varphi_{2,1} = 0.8;\ X_0 = 0;$

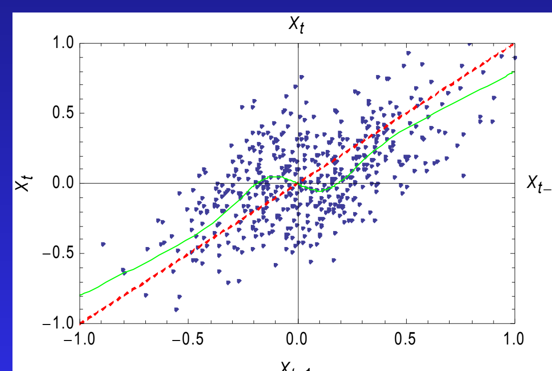


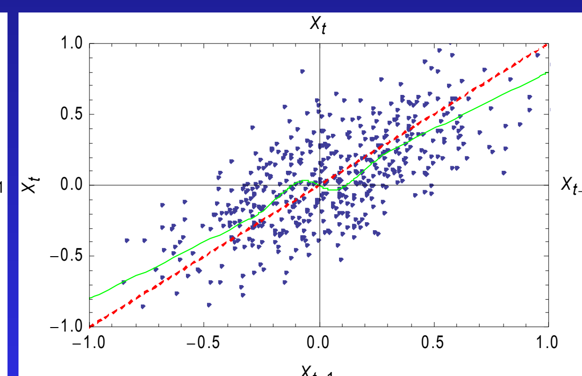




**$\gamma = 0$;** **$\gamma = 0.5$;** **$\gamma = 5$;**



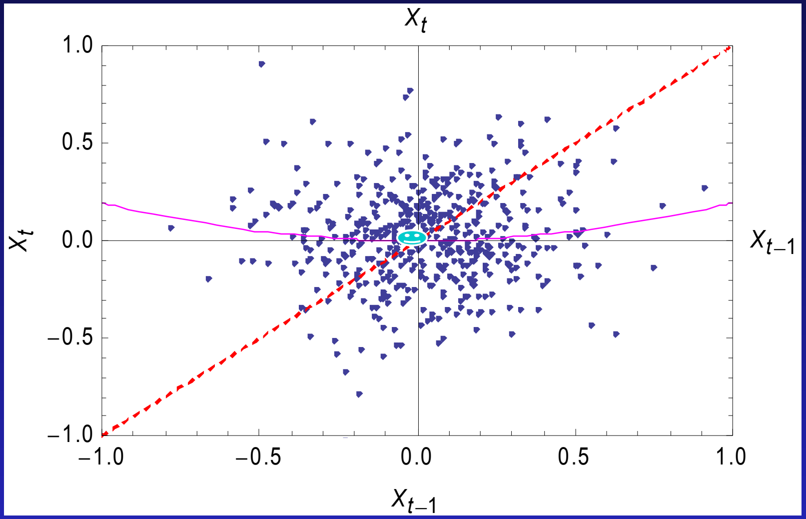



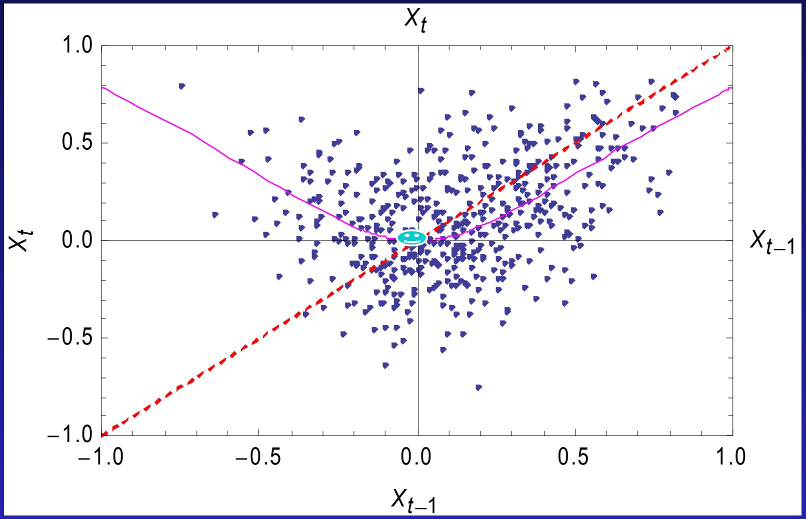

**$\gamma = 10$;** **$\gamma = 20$;** **$\gamma = 50$;**

$$X_t = (\varphi_{1,0} + \varphi_{1,1} X_{t-1})(1 - G(X_{t-1}, \gamma, c)) + (\varphi_{2,0} + \varphi_{2,1} X_{t-1}) G(X_{t-1}, \gamma, c) + \varepsilon_t$$

## LSTAR jedno stabilné ekvilibrium

LSTAR, $c = 0;\ \varphi_{1,0} = \varphi_{2,0} = 0;\ \ \varphi_{1,1} = -0.8;\ \ \varphi_{2,1} = 0.8;$



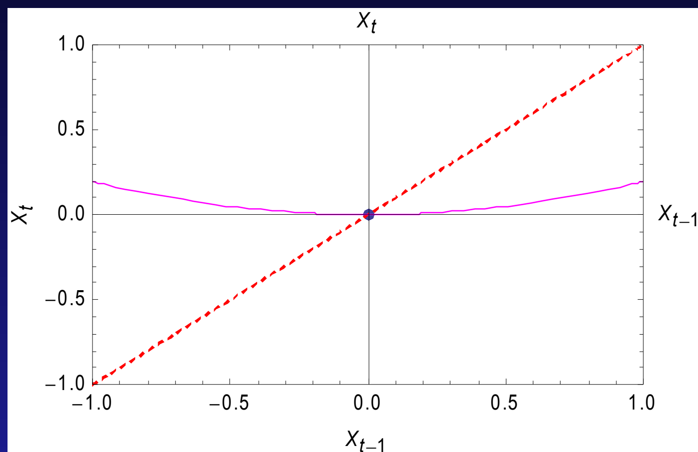



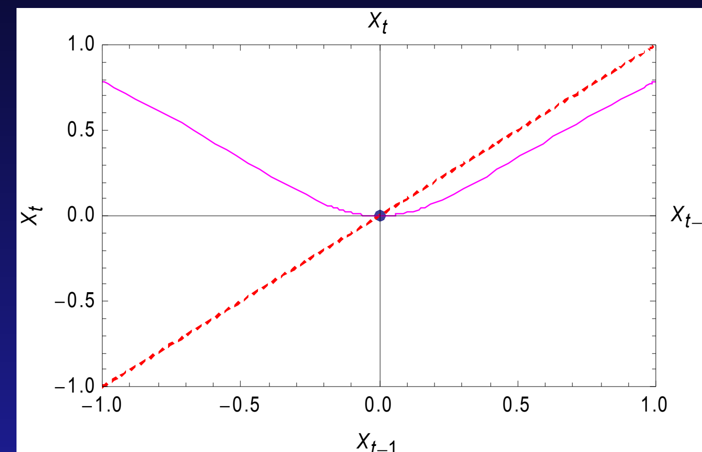

**$\gamma = 0.5$;** **$\gamma = 5$;**

Jediné stabilné equilibrium $X^* = 0$, $E[X_t] \neq 0$

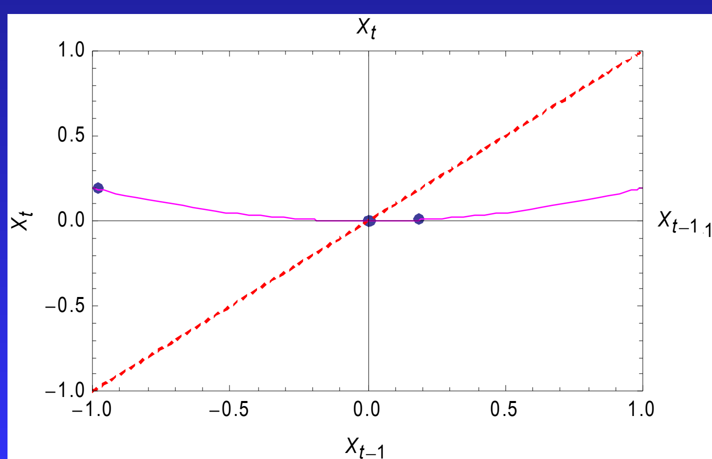
$$X_t = (\varphi_{1,0} + \varphi_{1,1} X_{t-1})(1 - G(X_{t-1}, \gamma, c)) + (\varphi_{2,0} + \varphi_{2,1} X_{t-1}) G(X_{t-1}, \gamma, c)$$

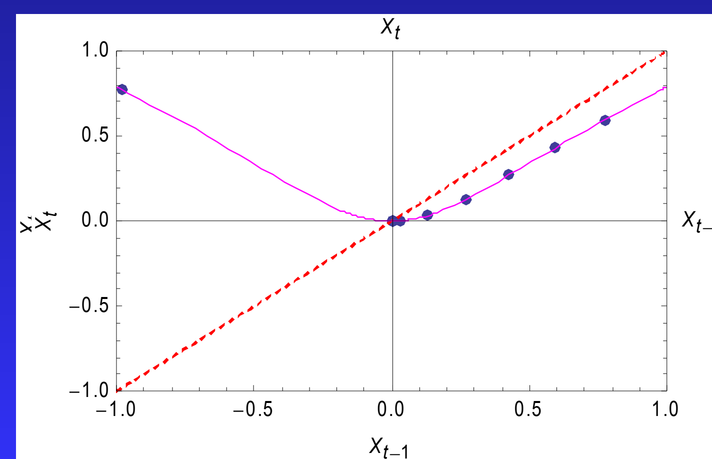
$c = 0; \quad \varphi_{1,0} = \varphi_{2,0} = 0; \quad \varphi_{1,1} = -0.8; \quad \varphi_{2,1} = 0.8; X_0 = 0$




$c = 0; \quad \varphi_{1,0} = \varphi_{2,0} = 0; \quad \varphi_{1,1} = -0.8; \quad \varphi_{2,1} = 0.8; X_0 = -1$

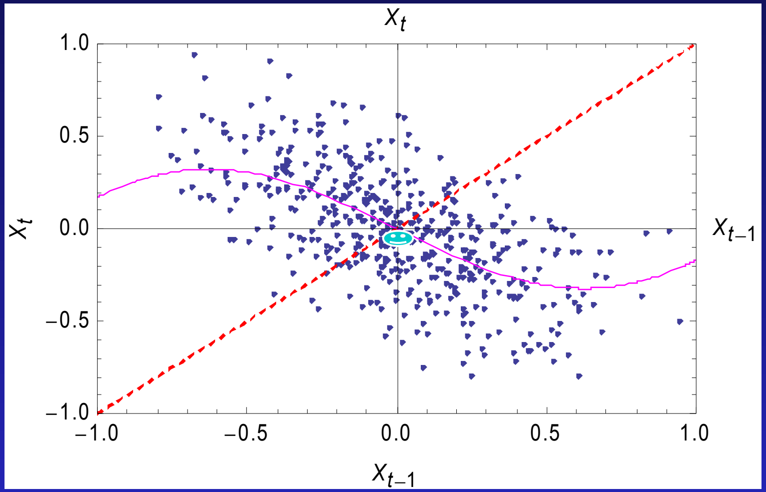


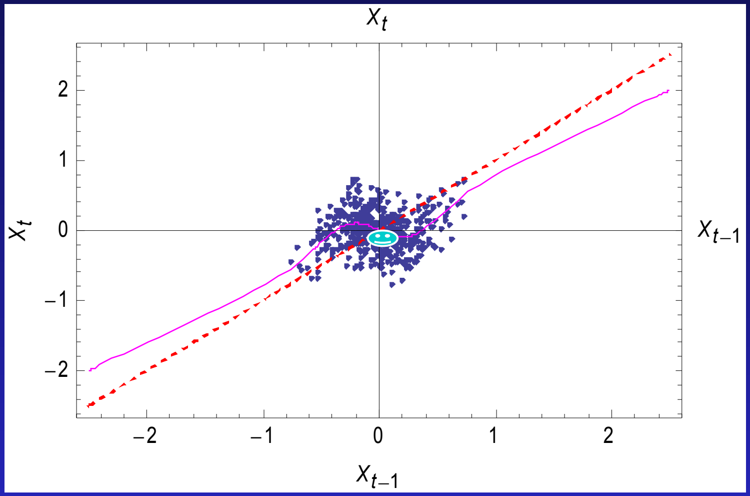



$$X_t = (\varphi_{1,0} + \varphi_{1,1} X_{t-1})(1 - G(X_{t-1}, \gamma, c)) + (\varphi_{2,0} + \varphi_{2,1} X_{t-1}) G(X_{t-1}, \gamma, c) + \varepsilon_t$$

# ESTAR jedno stabilné ekvilibrium

ESTAR, $c = 0$; $\varphi_{1,0} = \varphi_{2,0} = 0$; $\varphi_{1,1} = -0.8$; $\varphi_{2,1} = 0.8$;



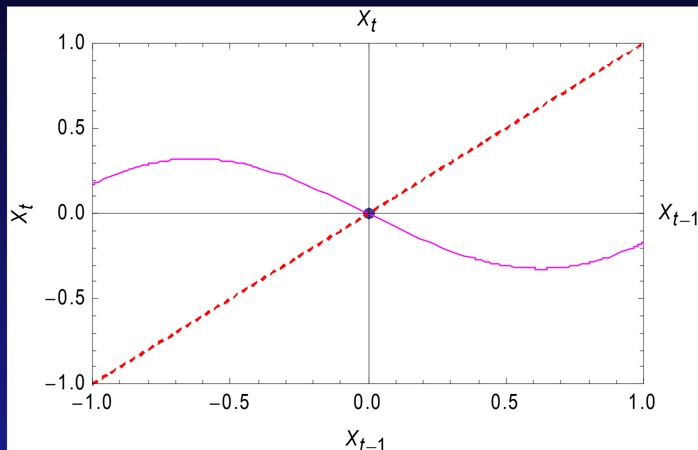



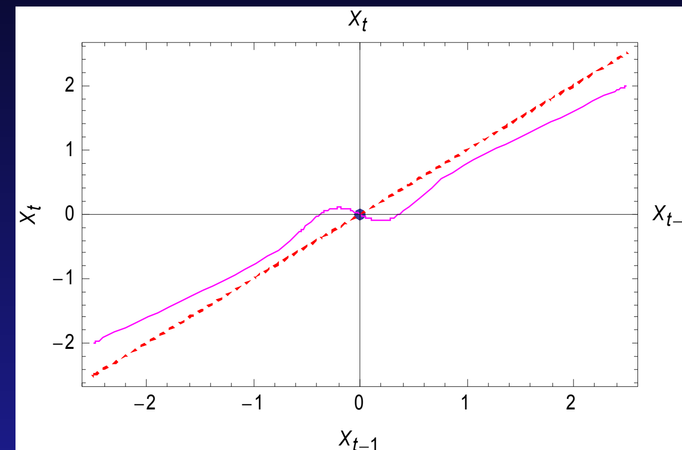

**$\gamma = 0.5$;** **$\gamma = 5$;**

Jediné stabilné equilibrium $X^* = 0$, $E[X_t] \neq 0$

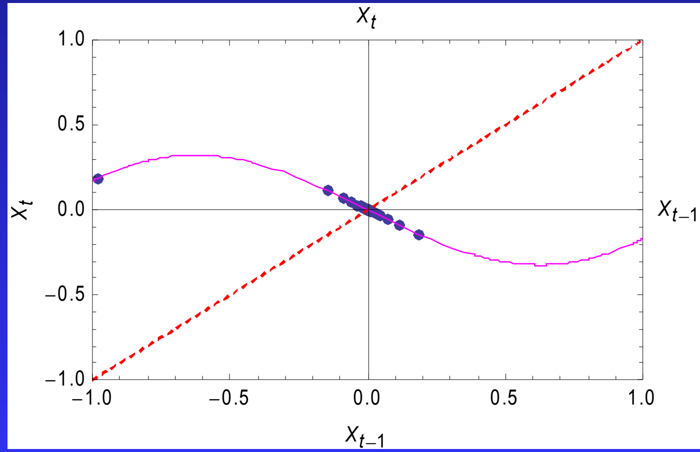
$$X_t = (\varphi_{1,0} + \varphi_{1,1} X_{t-1})(1 - G(X_{t-1}, \gamma, c)) + (\varphi_{2,0} + \varphi_{2,1} X_{t-1}) G(X_{t-1}, \gamma, c)$$

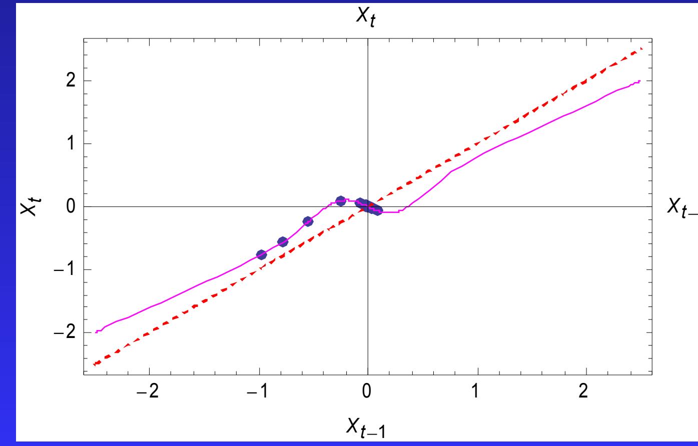
$c = 0$; $\varphi_{1,0} = \varphi_{2,0} = 0$; $\varphi_{1,1} = -0.8$; $\varphi_{2,1} = 0.8$; $X_0 = 0$




$c = 0$; $\varphi_{1,0} = \varphi_{2,0} = 0$; $\varphi_{1,1} = -0.8$; $\varphi_{2,1} = 0.8$; $X_0 = -1$

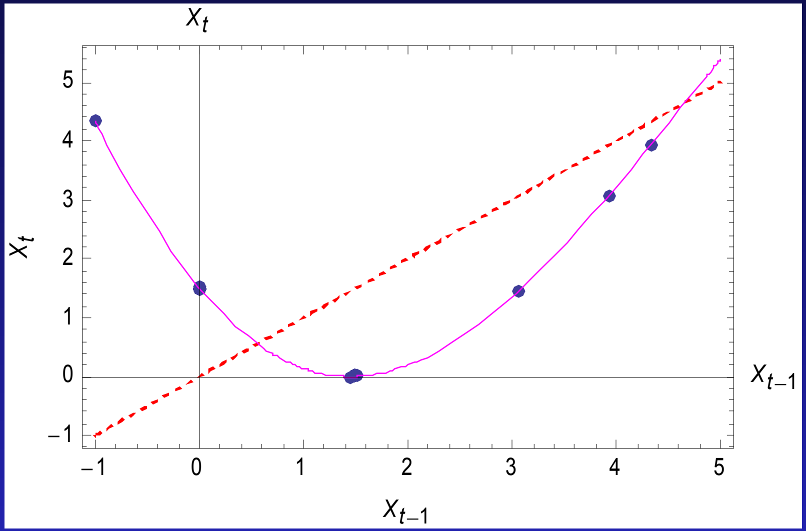


$$X_t = (\varphi_{1,0} + \varphi_{1,1} X_{t-1})(1 - G(X_{t-1}, \gamma, c)) + (\varphi_{2,0} + \varphi_{2,1} X_{t-1}) G(X_{t-1}, \gamma, c) + \varepsilon_t$$

## LSTAR dve nestabilné ekvilibria

$c = 0$; $\gamma = 0.5$; $\varphi_{1,0} = 6$; $\varphi_{2,0} = -3$; $\varphi_{1,1} = -4$; $\varphi_{2,1} = 2$;

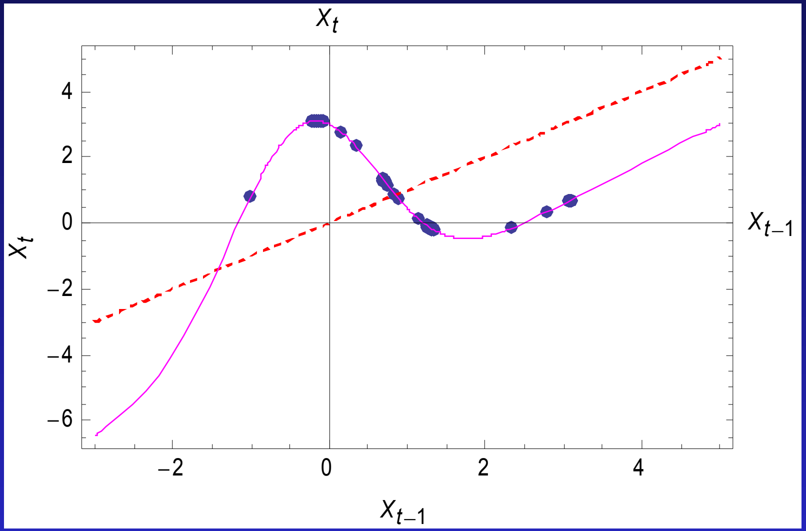



$X^{1,*} = 0.555$, $X^{2,*} = 4.677$

$$X_t = (\varphi_{1,0} + \varphi_{1,1} X_{t-1})(1 - G(X_{t-1}, \gamma, c)) + (\varphi_{2,0} + \varphi_{2,1} X_{t-1}) G(X_{t-1}, \gamma, c) + \varepsilon_t$$

## ESTAR tri nestabilné ekvilibria

$c = 0$; $\gamma = 0.5$; $\varphi_{1,0} = 3$; $\varphi_{2,0} = -3$; $\varphi_{1,1} = -1.1$; $\varphi_{2,1} = 1.2$;

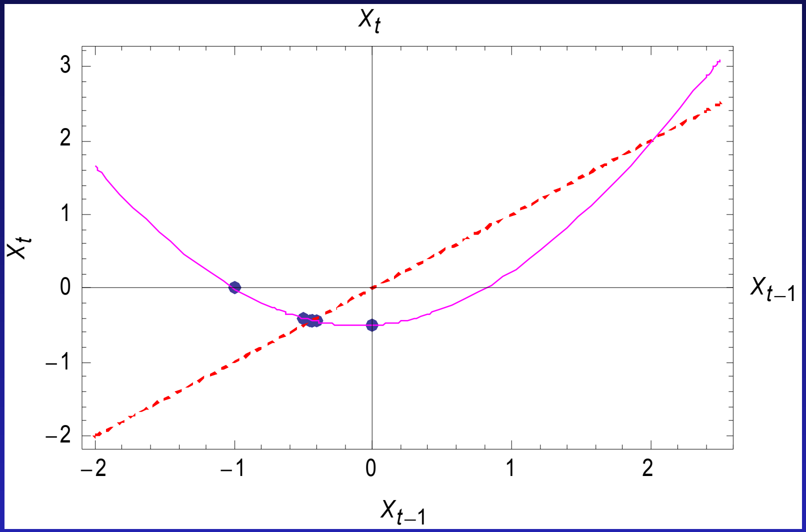



$X^{1,*} = -1.443$, $X^{2,*} = 0.847$, $X^{3,*} = 15$

$$X_t = (\varphi_{1,0} + \varphi_{1,1} X_{t-1})(1 - G(X_{t-1}, \gamma, c)) + (\varphi_{2,0} + \varphi_{2,1} X_{t-1}) G(X_{t-1}, \gamma, c) + \varepsilon_t$$

## LSTAR jedno stabilné, druhé nestabilné ekvilibrium

$c = 0$; $\gamma = 0.5$; $\varphi_{1,0} = -3$; $\varphi_{2,0} = 2$; $\varphi_{1,1} = -3$; $\varphi_{2,1} = 2$;

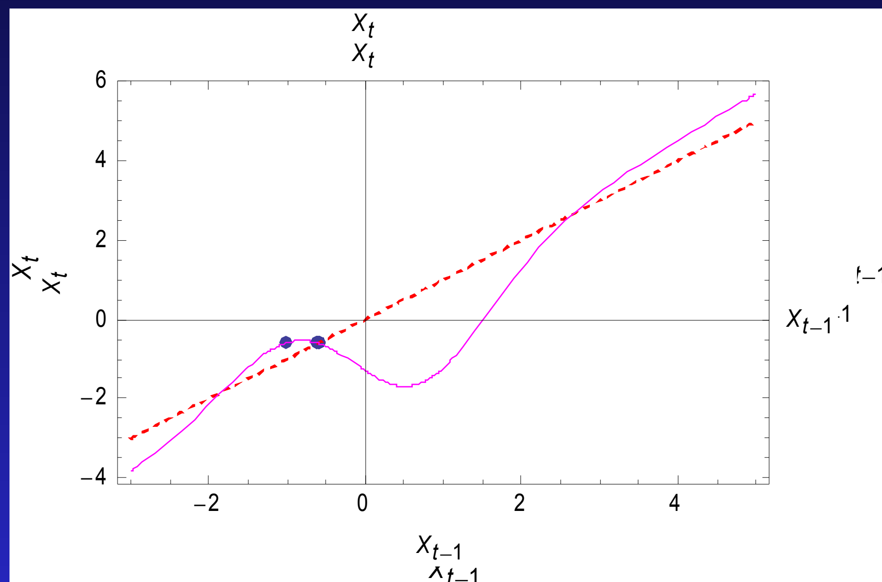



$X^{1,*} = -1.612$, $X^{2,*} = 5.298$

$$X_t = (\varphi_{1,0} + \varphi_{1,1} X_{t-1})(1 - G(X_{t-1}, \gamma, c)) + (\varphi_{2,0} + \varphi_{2,1} X_{t-1}) G(X_{t-1}, \gamma, c) + \varepsilon_t$$

**ESTAR 3 ekvilibria; jedno stabilné, dve nestabilné ekvilibrium**

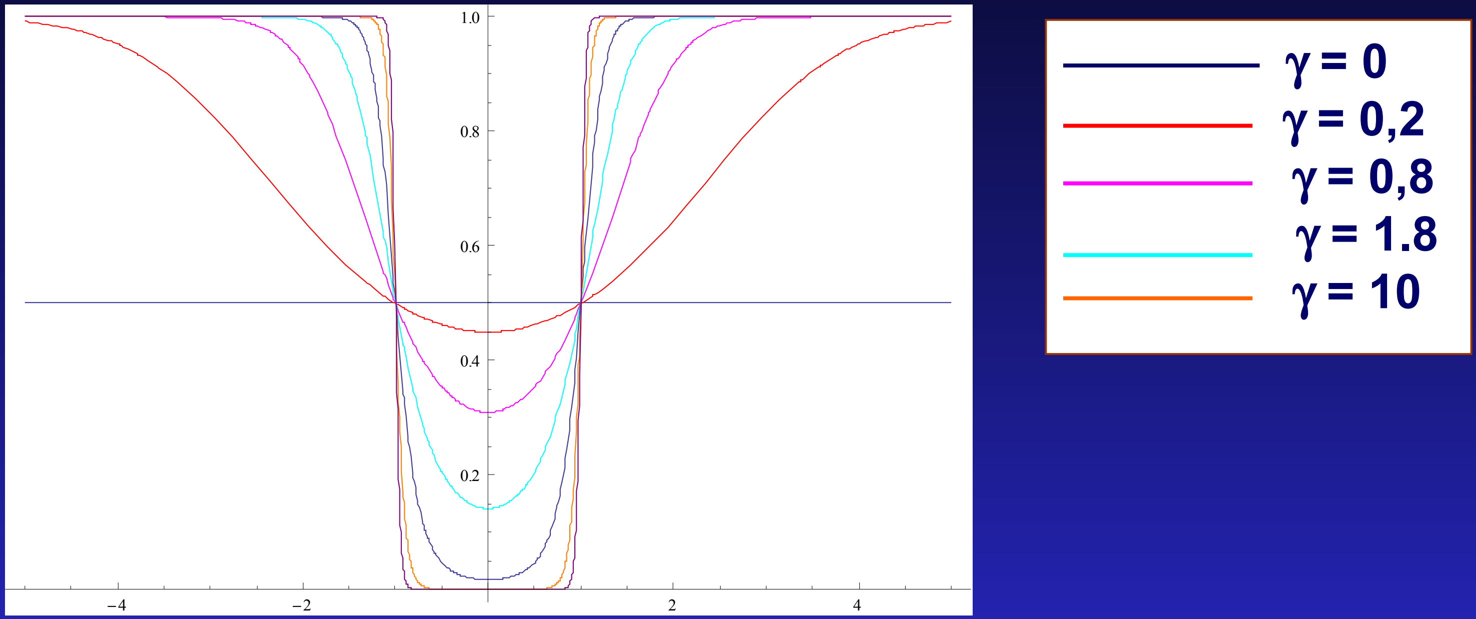
$c = 0$; $\gamma = 0.5$; $\varphi_{1,0} = -1.3$; $\varphi_{2,0} = -0.3$; $\varphi_{1,1} = -1.3$; $\varphi_{2,1} = 1.2$;




$X^{1,*} = -1.834$; $X^{2,*} = -0.602$, $X^{3,*} = 2.647$

# Logistická funkcia 2. stupňa (tri režimy)

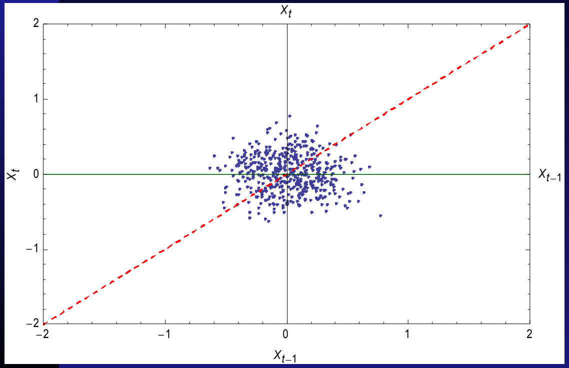
$$G(q_t,\gamma,c_1,c_2)=\frac{1}{1+e^{-\gamma(q_t-c_1)(q_t-c_2)}},\quad \gamma>0, c_1<c_2$$

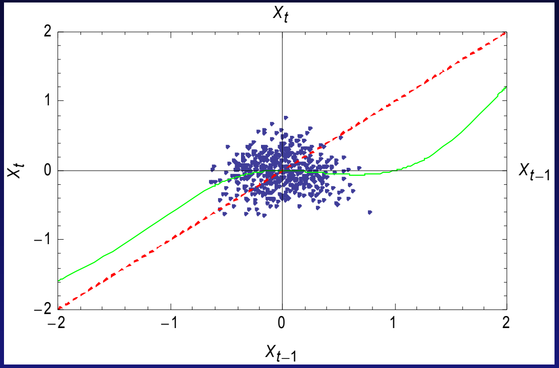


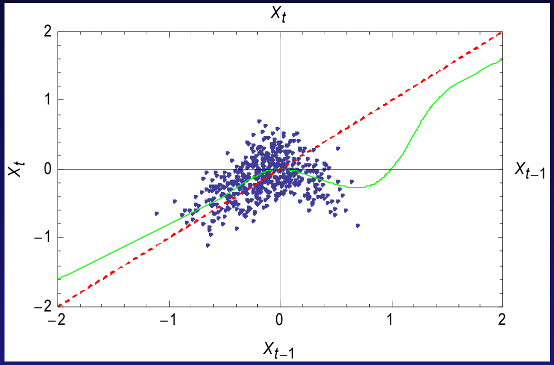

$c_1$ = -1, $c_2$ = 1

$$X_t = (\varphi_{1,0} + \varphi_{1,1} X_{t-1})\,(1 - G(X_{t-1}, \gamma, c_1, c_2)) + (\varphi_{2,0} + \varphi_{2,1} X_{t-1})\,G(X_{t-1}, \gamma, c_1, c_2) + \varepsilon_t$$

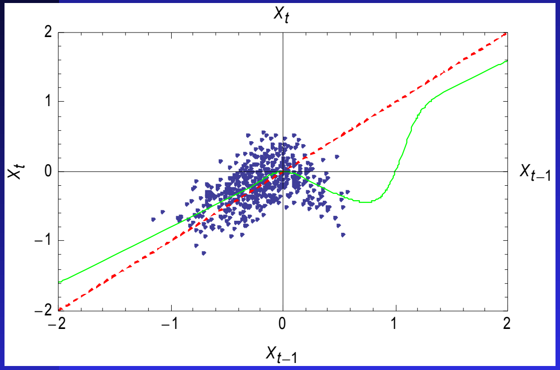
$c_1 = 0;\ c_2 = 1;\ \varphi_{1,0} = \varphi_{2,0} = 0;\ \ \varphi_{1,1} = -0.8;\ \ \varphi_{2,1} = 0.8;\ X_0 = 0;$

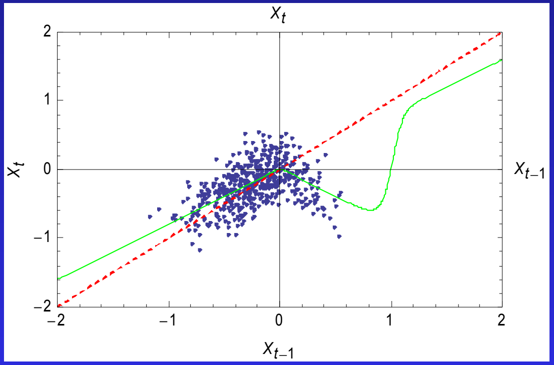


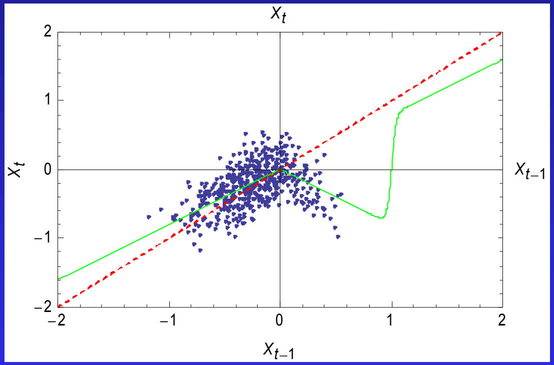




**γ = 0;** **γ = 0.5;** **γ = 5;**




**γ = 10;** **γ = 20;** **γ = 50;**

Za predpokladu *m* režimov definujeme *m* - 1 konštant $c_1, \dots, c_{m-1}$, pre ktoré platí:

$$c_1 < \dots < c_{m-1}$$

a *m* - 1 konštant $\gamma_1, \dots, \gamma_{m-1}$, pre ktoré platí $\gamma_i > 0$, i = 1, ..., *m* - 1.

Označme $p = \max(p_1, \dots, p_m)$,

$$Y_t = (1, X_{t-1}, \dots, X_{t-p})' \text{ a } \Phi_i = (\varphi_{i,0}, \varphi_{i,1}, \cdots, \varphi_{i,p})', i = 1, 2, \dots, m$$

$\{\varepsilon_t\}$ je i.i.d. proces s nulovou strednou hodnotou a rozptylom $\sigma_\varepsilon^2$.

m-režimový model LSTAR:

$$X_t = \Phi_1' \mathbf{Y}_t + (\Phi_2 - \Phi_1)' Y_t G(q_t, \gamma_1, c_1) + (\Phi_3 - \Phi_2)' Y_t G(q_t, \gamma_2, c_2) + \dots$$

$$+ (\Phi_m - \Phi_{m-1})' Y_t G(q_t, \gamma_{m-1}, c_{m-1}) + \varepsilon_t$$

# Osnova programu na štvrté cvičenie

**Diagnostická kontrola modelu SETAR:**

a) Testovanie ostávajúcej nelinearity

- Nahrať súbor LR_test.def a vlastný definičný súbor
- Pre najlepšie dvojrežimové modely:

1. odhadnúť parametre trojrežimového modelu (pre dané *p*)
2. vypočítať hodnotu autoregresných parametrov pre jednotlivé reži-my a pomocou informačnej matice vypočítať ich štandardné chyby
3. vypočítať reziduá trojrežimového modelu a ich rozptyl $S_3$
4. vypočítať P-value LR testu, kde prvý parameter je rozptyl reziduí dvojrežimového modelu $S_2$ a druhý rozptyl $S_3$

Poznámka: testovanie autokorelácie reziduí SETAR modelu urobíme dodatočne.